Sharing policies and resource provisioning in Grids

Prof. Costas Courcoubetis
Athens University of Economics and Business

joint work with
Prof. Richard Weber
Statistical Laboratory
Cambridge Univ.

Supported by FP6 project Gridecon

Outline

- Key questions
- Some gaming models for Grid participation
- Results and discussion
- Conclusions

Key questions

Given that Grid = resource pooling, is it always true that

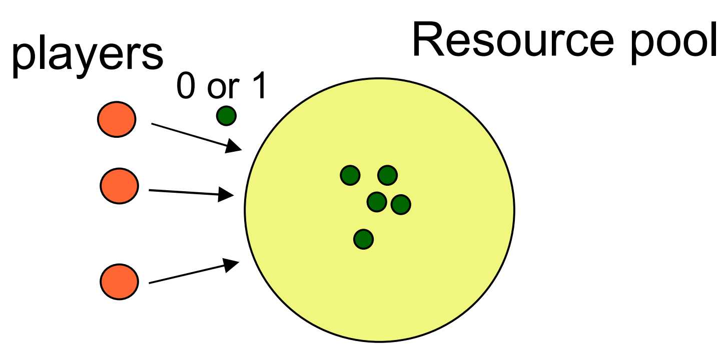
- Participating in a Grid is always better than self-provisioning?
  - Always gain from Grid participation?
- Sharing policies that maximize total performance should be preferred?
  - Egalitarian sharing vs prioritized? Internal pricing?
- How crucial are sharing policies for the sustainability of Grid infrastructures?
  - Stability issues?
- How to enable sequential participation?
  - Grid is build sequentially, different incentives/participant

The provisioning game

- Game context:
  - Phase 0: system designer posts policy for sharing
  - Phase 1: players decide on how much to contribute
  - Phase 2: the system operates according to posted policy where total resource = procured in phase 1, generates revenue to players
- Nash Equilibria (NE) in strategies of players
- Optimum centralized solution (CS)
  - buy resources centrally, charge participants
- Compute price of anarchy
  - How worse-off are the NE from the CS?
- Compute stand-alone cost for players
  - Are players better off by participating?

A simple model

- Game played in 3 phases
- Phase 0: the rules of resource sharing are posted
- Phase 1 : each player <u>chooses</u> his *strategy*: the probability to buy (or not) a unit of resource for the resource pool with cost per unit a
- Phase 2: each player <u>discovers</u> whether he needs 1 or 0 units of resource. This occurs with prob θ_i
  - If he needs but cannot get, costs him $c_i > a$



Complete sharing policy

Case of 2 symmetric players

$$a = 1, c = 4, \theta = .5$$

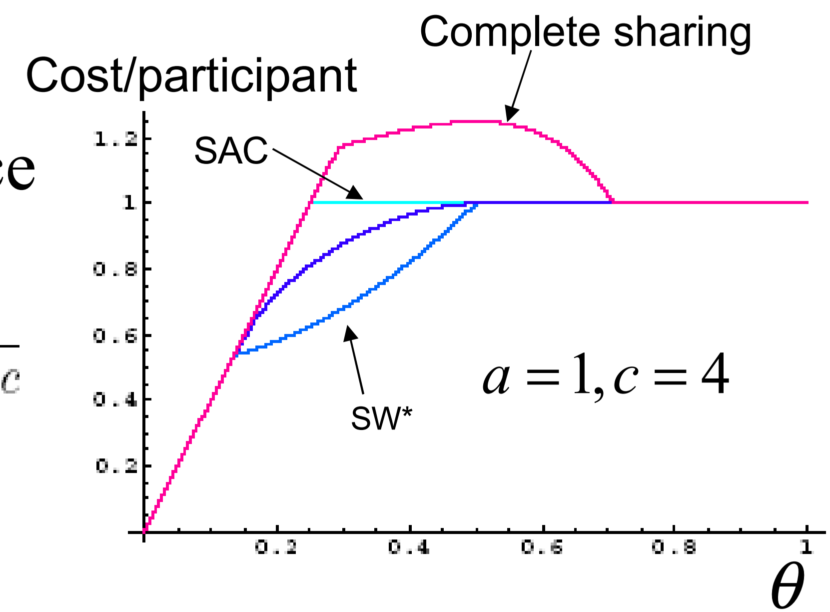
$$\begin{pmatrix} (a, a) & (a + \frac{1}{2}\theta^2 c, \frac{1}{2}\theta^2 c) \\ (\frac{1}{2}\theta^2 c, a + \frac{1}{2}\theta^2 c) & (\theta c, \theta c) \end{pmatrix} \qquad \begin{pmatrix} 1, 1 & 1.5, .5 \\ .5, 1.5 & 2, 2 \end{pmatrix}$$

Symmetric Nash equilibrium:

$p =$ prob to provide a unit of resource

$$p = \begin{cases} 0\,, & \theta \le 1 - \sqrt{1 - 2a/c} \\ 1 - \frac{2a - c\theta^2}{2c\theta(1-\theta)}\,, & 1 - \sqrt{1 - 2a/c} \le \theta \le \sqrt{2a/c} \\ 1\,, & \theta \ge \sqrt{2a/c} \end{cases}$$

$$a = 1, c = 4, \theta = .5 \Rightarrow p = .5$$



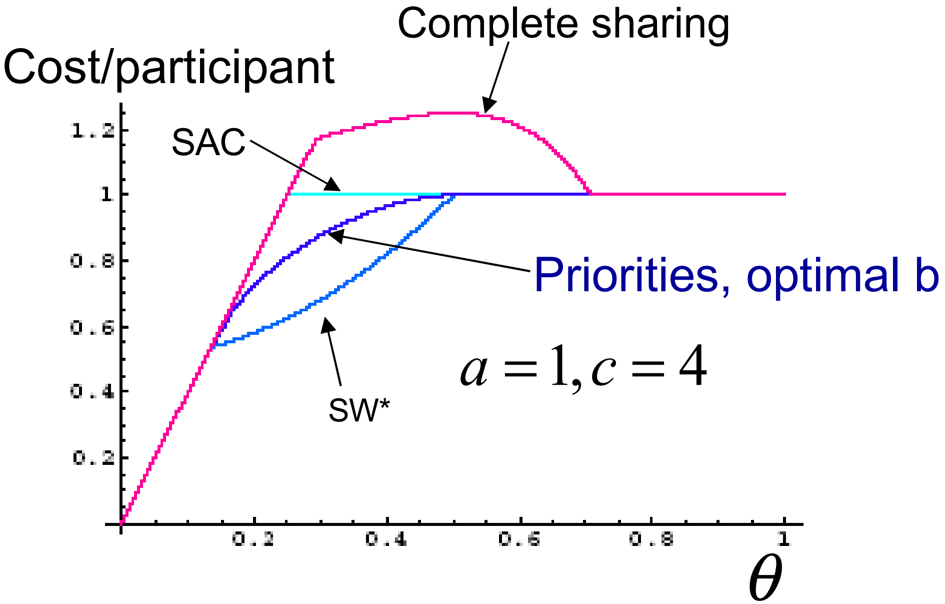
Results

- Players are worse-off if they participate!
  - Symmetric NE is worse than stand alone cost
  - Non-symmetric equilibria are unstable
  - n>2: symmetric solution gets worse!
- Optimal central planning: provide k=0,1,or 2 units
  - k=1: Impossible to achieve using a symmetric policy!
- Can we do any better?
  - Choose other sharing policy!

Priority in sharing

- A player that contributes has priority
- If he does not need the resource, charges b other players

$$\begin{pmatrix} (a,a) & (a-\theta(1-\theta)b, \theta(1-\theta)b+\theta^2 c) \\ (\theta(1-\theta)b+\theta^2 c, a-\theta(1-\theta)b) & (\theta c, \theta c) \end{pmatrix}$$



A continuous model

- Player i contributes x_i, uses X_i, obtains utility $u_i(X_i)$
- Obtains net benefit $\theta_i E u(X_i) - a x_i$
- Has complete control on his contribution, may get more
- Expected utility of player i =
  - Extra capacity allocated equally

$$\theta_i E\left[u\left(x_i + \frac{\sum_j (1-I_j)x_j}{\sum_j I_j}\right) \middle| I_i = 1\right] - a x_i$$

  - Extra capacity allocated in proportion of contributions

$$\theta_i E\left[u_i\left(x_i + \frac{x_i}{\sum_j x_j I_j}\sum_j (1-I_j)x_j\right) \middle| I_i = 1\right] - a x_i$$

$I_j = 1$ if player j requests resources

Initial results

- Equal sharing of excess capacity: may get unstable symmetric NE
- Sharing excess capacity in proportion of contributions: stable NE
- Policy may influence stability besides efficiency!
- Optimal sharing may not be optimal overall
  - Equal sharing of available capacity is not recommended!

Sequential games

- Players sequentially decide whether or not to join a Grid facility
- A player chooses how much to contribute based on known (and anticipated) contributions of previous (and subsequent) players
- Interesting questions:
  - Which players will contribute? Who is better off? joining early or later?
  - How does the result compare with the simultaneous game?
  - Which policies maximize final result? Incentives for joining may depend on the number of players that joined already

Preliminary results

- When players are identical then early joiners will be the ones who contribute nothing
- However, if we use cross-payments (via b) then it can be made that the resource pool ends up at a size where the efficiency of the centralized solution is obtained

Conclusions

- Sustainability of Grid infrastructures is related to the efficiency and stability of games where players maximize their net benefit
- Sharing policies seem to influence efficiency and stability by determining the size of the system to be shared
- Optimal sharing may not be optimal overall
- How to design such optimal policies: hard problem, needs more work, analysis offers only some insights
- Sequential participation raises more interesting issues
- Analyze partial information models (unknown θ_i, c_i)